arXiv:hep-th/0405193v1 23 May 2004

Plane Wave Type II* String Backgrounds

Harvendra Singh[*]

Department of Physics,
Indian Institute of Technology, Guwahati-781039, Assam, India

ABSTRACT: In this note we aim to study plane-wave limits of the solutions of type II* superstring theories. We consider Freund-Rubin type $dS_5 \times H^5$ solutions of type IIB* theory and obtain a new kind of plane-wave solutions, we refer them as de Sitter plane-waves or Dpp-waves. Considering Hull's time-like T-duality we are able to map the Dpp wave solution to maximally supersymmetric Hpp-wave in IIB string theory and vice-versa.

KEYWORDS: Penrose Limits, Strings, Plane-Waves.

[*]hsingh@iitg.ernet.in

Contents

1. Introduction

The supersymmetric plane-wave (pp-wave) backgrounds in string theory [1, 2] could, generically, be obtained by applying the Penrose-like limits [3, 4, 2] on anti de Sitter spacetimes, $AdS \times Sphere$. Under these plane-wave limits [2] we basically zoom in onto those null geodesics which have a direction along the sphere. If the null geodesic is chosen such that it does not have a component along sphere then we obtain flat-space solutions. Although, the pp-waves are (asymptotically) non-flat geometries, nevertheless string theory in these backgrounds becomes exactly solvable theory in suitable light-cone gauge [5, 6]. Therefore, from AdS/CFT correspondence point of view [7], a pp-wave spacetime in bulk has useful consequences for dual conformal field theories on the boundary [8]. Under BMN-correspondence [8] the conformal field theory operators with large $U(1)$ R-charge are dual to type IIB closed string excitations in a pp-wave background spacetime. In this work we would like to concentrate on the plane-wave geometries in type II* string theories [9].

It was proposed some time back by Hull [9] that if we consider time-like T-duality of usual type II string theory with spacetime signature $(1, 9)$, then one could study string theories on arbitrary (m, n) signature spacetimes as well. This duality along time direction some how takes away the significance of the spacetime signature from the string theory, in particular the significance of Lorentzian symmetry $SO(1, 9)$. However, while implementing time-like duality it requires the assumption

of a compact time coordinate. It also makes the RR-potential fields of the dual theory tachyonic (ghost-like) in nature, that is, the kinetic terms with negative sign appear in the low energy effective action. Such type II string theories with tachyonic kinetic terms have been designated as II* string theories [9]. Nevertheless, these star-theories are maximally supersymmetric and hopefully the quantum fluctuations cancel out in the spectrum which could bring stability to the flat spacetime which is still the vacuum solution. We note that the theories with tachyonic matter fields or with a tachyonic potential are favourable candidates for cosmology, see [10] and references therein, so that we are able to obtain de-Sitter spacetime. The de Sitter spacetimes naturally appear as solutions in II* theories. The usual string theory forbid de-Sitter space as a classical solutions, but see [12] for recent developments.

This paper is organised as follows. We first discuss the Penrose limits in the case of type II* supergravity in section-2. Then we consider de Sitter solution of type IIB* supergravity and apply plane-wave limits to obtain the new kind of de Sitter plane-wave (Dpp) background which can exist only in type II* string theory. In section-3 we discuss time-like T-duality and the relationship between Hpp and Dpp waves. The conclusions are given in section-4.

2. A case for de-Sitter plane-wave (Dpp-wave)

2.1 Review: The Penrose Limits

The plane-wave limits of anti-de Sitter spacetime, $AdS_p \times S^q$, along a null geodesic with a non-zero component along the sphere leads to a plane-wave geometry. The procedure is well described in [2] where maximally supersymmetric Hpp-wave solutions are obtained by taking the limits of $AdS_5 \times S^5$ type IIB background. Our interest here is to study plane-wave limits for the solutions of type II* superstring theories for which there exist de Sitter solutions.

In type II* supergravity [9], the action has the kinetic terms for all the RR-potentials with wrong sign, so they are tachyonic. Originally these theories are obtained by implementing T-duality along compact time-like directions of ordinary type II string theory. Assuming a compact time like direction in a theory is pathological, but after the transformations these tachyonic string theories have usual Minkowskii signature (1,9) and the maximal supersymmetry.[1] It is straightforward and obvious that II*-string theories will have following scaling limits on the fields

$$g_{\mu\nu} \to \xi^{-2} g_{\mu\nu}, \quad \phi \to \phi, \quad A_{(p)} \to \xi^{-p} A_{(p)}. \tag{2.1}$$

---

[1]The assumption of a compact time direction is considered only as a tool to implement time-like duality [9]. However, the RR-sector kinetic energy terms having minus sign is somewhat troublesome. But it is nevertheless hoped that the supersymmetry takes care of these instabilities since the dual string theory is well defined.

under which the action scales homogeneously. The scale parameter ξ has to be strictly positive. [2]

The Penrose limits are of the usual type

$$\bar{g}_{\mu\nu} = \lim_{\Omega \to 0} \Omega^{-2} g_{\mu\nu}(\Omega)$$
$$\bar{\phi} = \lim_{\Omega \to 0} \phi(\Omega)$$
$$\bar{A}_{(p)} = \lim_{\Omega \to 0} \Omega^{-p} A_{(p)}(\Omega) \tag{2.2}$$

where parameter Ω is positive. Now the proposal is that if there exists a (anti) de-Sitter solution with the local data $((A)dS, g, \phi, A_p)$, by implementing the limits (2.2) we will get to new data $(PP, \bar{g}, \bar{\phi}, \bar{A}_p)$ which is a Hpp (Dpp)-wave of type IIB (IIB*) string theory. In the next secion we will explicitly show this by taking an example of de-Sitter background of IIB* supergravity, the other case of Hpp-wave is by now well studied [2]. It is worth noting that the limits (2.2) are the same as in Blau et al [2].

2.2 The $dS_5 \times H^5$ and a Dpp-wave solution

The type IIB* supergravity [9] the action has the kinetic terms of all the RR-potentials with wrong sign, see Appendix. We shall assume that the IIB* theory has a stable vacuum and is a consistent string theory. Nevertheless there exists a de-Sitter solution $dS_5 \times H^5$ [14, 15]. This de Sitter solution is given by

$$g := l^2 \{ -dt^2 + cosh^2 t \left(\frac{dr^2}{1-r^2} + r^2 d\Omega_3^2 \right) \} + l^2 \{ d\psi^2 + cosh^2 \psi \left(\frac{ds^2}{1+s^2} + s^2 d\Omega_3^2 \right) \}$$
$$F_{(5)} = 2\sqrt{2}\, l^4 (\mathrm{Vol}(dS_5) + \mathrm{Vol}(H_5)), \tag{2.3}$$

where l represents the radii of the de-Sitter dS_5 and hyperbolic space H^5, and $d\Omega_n^2$ is the line element of unit n-sphere. The volume 5-forms are defined for the unit size five-spaces, e.g.,

$$\mathrm{Vol}(dS_5) = cosh^4 t \frac{r^3}{\sqrt{1-r^2}} dt dr \omega_3 \ ,$$

where ω_3 is the volume form over unit radius 3-sphere. The five-form field strength in (2.3) is such that it is self-dual. The rest of the background fields like dilaton etc. are vanishing in (2.3).

As usual in the case of plane-wave limits, we would like to change the coordinates in the (ψ, t) plane to

$$U = \psi + t, \quad V = \psi - t, \tag{2.4}$$

---

[2]We remark that such scaling symmetry of action could be used to tune the mass scales, if any, in a theory like in massive/gauged type IIA, see [13].

in terms of which the background becomes,

$$l^{-2}g := dUdV + cosh^2(\frac{U-V}{2})\left(\frac{dr^2}{1-r^2} + r^2d\Omega_3^2\right) + cosh^2(\frac{U+V}{2})\left(\frac{ds^2}{1+s^2} + s^2d\Omega_3^2\right)$$

$$l^{-4}F_{(5)} = 2\sqrt{2}\{\text{Vol}(dS_5(U,V)) + \text{Vol}(H_5(U,V))\}, \tag{2.5}$$

We shall now rescale the coordinates as

$$U = u, \quad V = \frac{v}{(l)^2}, \quad Y^a = \frac{y^a}{l}, \quad Y^\alpha = \frac{y^\alpha}{l} \tag{2.6}$$

where $r^2 = Y^aY^a$, $s^2 = Y^\alpha Y^\alpha$. Then we take the limits (2.2), $l \to \infty$, i.e. large radius limit, along the null geodesic parametrised by U.[3] This consists in dropping the dependence on the coordinates other than u. In this way we get the pp-wave solution written in Rosen coordinates and depending only on u,

$$\bar{g} := dudv + cosh^2(\frac{u}{2})\sum_{a=1}^{4}(dy^a)^2 + cosh^2(\frac{u}{2})\sum_{\alpha=5}^{8}(dy^\alpha)^2$$

$$\bar{F}_{(5)} = 2\sqrt{2}\;\{cosh^3(\frac{u}{2})\;dudy^1dy^2dy^3dy^4 + cosh^3(\frac{u}{2})\;dudy^5dy^6dy^7dy^8\}. \tag{2.7}$$

In order to write the above background in the familiar form, we shall like to switch to the Brinkman coordinates (dx^+, dx^-, x^a) as in [1], but these transformations are slightly different. It is not difficult to guess that the new coordinate relations are

$$x^- = u/2, \quad x^+ = v - \frac{1}{4}\sum_{i=1}^{8}\frac{sinh(2\lambda_i u)}{2\lambda_i}y^iy^i, \quad x^i = y^i\frac{cosh(\lambda_i u)}{2\lambda_i}. \tag{2.8}$$

After some straightforward steps we obtain the Dpp-wave metric in Cahen-Wallach spacetime form

$$g := 2dx^+dx^- + (A_{ab}x^ax^b)(dx^-)^2 + \sum_{a=1}^{8}(dx^a)^2$$

$$F_{-1234} = 2\sqrt{2} = F_{-5678} \tag{2.9}$$

with the matrix

$$A_{ab} \equiv \delta_{ab}. \tag{2.10}$$

It can be calculated that the only nonvanishing componont of the Ricci tensor for the Dpp-wave metric (2.9) is

$$R_{--} = -8 ,$$

which is strictly negative.[4] Also note that the matrix A in (2.10) is diagonal and positive which is opposite of the case of Hpp-wave [1]. It is obvious because the background (2.9) satisfies the field equations of type IIB* supergravity where $C_{(4)}$ kinetic term has opposite sign unlike in ordinary IIB theory.

[3]These limits involve zooming in onto a null geodesic with component along H^5, the limits basically are the Penrose limits in [2].

[4]We follow the convention where for a de Sitter space of radius of curvature l, the Ricci tensor is given by $R_{\mu\nu} = \frac{(D-1)}{l^2}g_{\mu\nu}$. Here D is the spacetime dimension.

3. The time-like T-duality

3.1 Dpp-wave from Hpp-wave

The time-like T-duality map between the fields of two types of strings has been constructed by Hull [9]. Under this map the background fields of type IIB (IIA) string theory are mapped to the fields of IIA* (IIB*) theory and vice-versa. Consider the Hpp-wave [1]

$$g := 2dx^+ dx^- + (H_{ab} x^a x^b)(dx^+)^2 + \sum_{a=1}^{8} (dx^a)^2$$

$$F_{(5)} = 2\sqrt{2}\mu \ dx^+ (dx^1 dx^2 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8) \tag{3.1}$$

with the matrix $H_{ab} = -\mu^2 \delta_{ab}$. The Ricci tensor for Hpp-wave is

$$R_{--} = 8\mu^2 \ ,$$

which is positive unlike the Dpp-wave. In (3.1) we can consider x^+ as time coordinate while x^- acts like a space-like direction. So the 5-form flux is electric type. We will take x^+ coordinate of (3.1) and make a time-like duality [9]. Using the duality relations given in the Appendix, the background we obtain as a result is IIA* extended string solution

$$g := W^{-1} \left\{ -(dx^+)^2 + (dx^-)^2 \right\} + \sum_{a=1}^{8} (dx^a)^2$$

$$F_{(4)} = 2\sqrt{2}\mu \ (dx^1 dx^2 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8)$$

$$B_{(2)} = -W^{-1} dx^+ dx^-, \quad e^{2\phi} = W^{-1} \tag{3.2}$$

where B is the NS-NS field under which fundamental strings are charged with and the function $W = \mu^2 \delta_{ab} x^a x^b$. $F_{(4)}$ is the tachyonic R-R four-form field strength. It is interesting at the first place to note that such a nontrivial string-like solution, in presence of tachyonic RR-fluxes, exists for IIA* theory action (A.1). We, though, make it clear that an extended fundamental string solution [11] when tachyonic RR backgrounds are vanishing is anyway a solution of II* theories.

In the next step we make a space-like T-duality by compactifying the space like direction x^- of the background (3.2). This gives us a type IIB* solution

$$g := 2dx^+ dx^- + (\mu^2 \delta_{ab} x^a x^b)(dx^-)^2 + \sum_{a=1}^{8} (dx^a)^2$$

$$F_{(5)} = 2\sqrt{2}\mu dx^- (dx^1 dx^2 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8), \tag{3.3}$$

which is nothing but the Dpp-wave given in (2.9) when $\mu = 1$. In eq.(3.3) the coordinate x^- behaves like a space direction, the flux $F_{(5)}$ is magnetic type. Hence

the roles of x^+ and x^- have got exhanged under the above duality map which includes one time-like T-duality.

Thus we see that starting from Hpp-wave of IIB string and implementing a time-like duality and a space-like duality in succession, we have obtained Dpp-wave which is a solution of the IIB* string theory. This is confirmation of the fact that the Hull's time-like duality map works for the case of plane-waves.

The amount of supersymmetries preserved by the Dpp-wave solution (2.9) is not clear as it is not straight forward to work out the Killing spinors due to missing ingredients like supersymmetry variations of the II* theories. But as we know type II* string theories have 32 supersymmetries [9]. So we would like to claim that the supersymmetry preserved by the Dpp-wave solutions is the same as those in Hpp-wave, which are maximally supersymmetric plane-wave solutions of IIB strings. The main reason behind this proposal is that the Hpp and Dpp are time-like T-dual backgrounds as we have seen above. If the time-like T-duality preserves the supersymmetry of the background then Dpp-waves are maximally supersymmetric solutions.

3.2 M-theory on $T^{1,1}$

As we know M-theory compactification on a spatial 2-torus, $T^{0,2}$, in the limit of shrinking torus leads to type IIB string theory. Similarly, M-theory compactification on a Lorentzian torus, $T^{1,1}$, in the limit of torus shrinking to zero size leads to type IIB* strings [9]. So there is way to expect that the Dpp-wave is related to a solution of M-theory. On the other hand, one can also do the oxidation of the type IIA* string solution (3.2) to eleven dimensions which will give us following $(9 + 2)$-dimensional M* theory solution [9]

$$g_{(11)} := W^{-\frac{2}{3}} \left\{ -(dx^+)^2 + (dx^-)^2 - dz^2 \right\} + W^{\frac{1}{3}} \sum_{a=1}^{8} (dx^a)^2$$

$$F_{(4)} \equiv dC_{(3)} = W^{-2}(\partial_a W)dx^a dx^+ dx^- dz + 2\sqrt{2}\mu(dx^1 dx^2 dx^3 dx^4 + dx^5 dx^6 dx^7 dx^8)$$

$$(3.4)$$

where W is as in (3.2). The coordinates x^+, z are two time like directions in M* theory. It is useful to mantion that the low energy supergravity action for M* theory of which eq.(3.4) is a background solution, has negative kinetic terms for 3-form potential $C_{(3)}$ [9]. The solution (3.4) has both electric as well as magnetic type fluxes, hence it can be described as a bound state of 2-branes and 5-branes of M* theory.

4. Conclusions

We have first shown that the Dpp-wave (2.9) is a solution of the IIB* string theory which can be obtained by taking pp-wave limits of the corresponding de Sitter

solution. These Dpp-waves are of a new kind in that they have the properties

$$R_{--} < 0$$

and entries of the matrix A_{ab} are strictly positive. These values are just opposite in sign compared to those of Hpp-waves. Then we have shown that starting from Hpp-wave of IIB string theory and by implementing a time-like and a space-like T-duality in succession, we obtain nothing but the Dpp-wave background. This is confirmation of the fact that the Hull's time-like duality map works for the case of Hpp-waves. It will be interesting to study time-like duality maps for other pp-wave backgrounds of string theory.

This study also provides us with a classfication of plane waves. Precisely we have got three types of plane-wave space-times; viz, the *Hpp-waves* [1]

i) $R_{--} > 0$ with $A_{ab} < 0$, $Tr A_{ab} \neq 0$,

the *plane-waves without matter* [16]

ii) $R_{--} = 0$ with $Tr A_{ab} = 0$

and the IIB* *Dpp-waves* with

iii) $R_{--} < 0$ with $A_{ab} > 0$, $Tr A_{ab} \neq 0$.

It would be worthwhile to study the BMN-correspondence for the case of Dpp-waves. Though at the first place it is not clear what is the exact nature of dS/CFT correspondence for de Sitter backgrounds [17], which are natural solutions of type II* string theories.

Acknowledgements

A. <u>Action in type II* supergravity</u>

The type IIA* effective action with $(1,9)$ spacetime signature is given by [9]

$$S = \int \left[e^{-2\phi} \left\{ R \, {}^*1 + 4 d\phi \, {}^*d\phi - \frac{1}{2} H_{(3)} \, {}^*H_{(3)} \right\} + \frac{1}{2} F_{(2)} \, {}^*F_{(2)} + \frac{1}{2} F_{(4)} \, {}^*F_{(4)} \right.$$
$$\left. + \frac{1}{2} dC_{(3)} dC_{(3)} B_{(2)} + \cdots \right], \tag{A.1}$$

where all Ramond-Ramond kinetic terms are ghost-like. The field strengths are defined through $F_{(n)} = dC_{n-1} + \cdots$. It is the same thing for type IIB* string theory

$$S = \int \left[e^{-2\phi} \left\{ R \, {}^*1 + 4 d\phi \, {}^*d\phi - \frac{1}{2} H_{(3)} \, {}^*H_{(3)} \right\} + \frac{1}{2} F_{(1)} \, {}^*F_{(1)} + \frac{1}{2} F_{(5)} \, {}^*F_{(5)} + \cdots \right], \tag{A.2}$$

for which equations of motion are supplemented with the constraint equation that five-form field strength satisfies self-dual equation $F_{(5)} =^* F_{(5)}$, where * is the Hodge-dual in ten dimensions.

For a time-like T-duality in the X^0 direction the relations between dual [9] backgrounds are

$$\tilde{g}_{00} = \frac{1}{g_{00}}$$

$$\tilde{g}_{0\alpha} = \frac{B_{0\alpha}}{g_{00}}$$

$$\tilde{B}_{0\alpha} = \frac{g_{0\alpha}}{g_{00}}$$

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} - \frac{g_{0\alpha}g_{0\beta} - B_{0\alpha}B_{0\beta}}{g_{00}}$$

$$\tilde{B}_{\alpha\beta} = B_{\alpha\beta} - \frac{g_{0\alpha}B_{0\beta} - B_{0\alpha}g_{0\beta}}{g_{00}}$$

$$\tilde{\phi} = \phi - \frac{1}{2}log|g_{00}| \tag{A.3}$$

along with the RR-fields strengths F_n of the type IIA (IIB) theory related to the field strengths $\tilde{F}_{n\pm1}$ of the dual type IIB* (IIA*) theory as

$$\tilde{F}_{0\mu_1\cdots\mu_n} = F_{\mu_1\cdots\mu_n}, \quad \tilde{F}_{\mu_1\cdots\mu_n} = -F_{0\mu_1\cdots\mu_n}$$

for any $\mu_i \neq 0$.

References

[1] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, JHEP 0201 (2002) 047, `hep-th/0110242`.

[2] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, `hep-th/0201081`;
J. Figueroa-O'Farrill and G. Papadopoulos, *Maximally Supersymmetric Solutions of Ten- and Eleven-dimensional Supergravities*, hep-th/0211089.

[3] R. Penrose, Any spacetime has a plane wave as a limit, in *Differential Geometry and relativity*, pp.271-75, Reidel, Dordrecht, 1976.

[4] R. Güven, *Plane wave limits and T-duality*, Phys. Lett. **B 482** (2000) 255, `hep-th/0005061`.

[5] R.R. Matsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, Nucl. Phys. **B 625** (2002) 70, `hep-th/0112044`.

[6] R.R. Matsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plave wave Ramond-Ramond backgrond*, `hep-th/0202109`.

[7] For a review see; O. Aharony, S.S. Gubser, J. Maldacena, H. Ooguri and Y. Oz, *Phys. Rep.* **323** (2000) 183, `hep-th/9905111`.

[8] D. Berenstein, J. Maldacena and H. Nastase, *Strings in flat space and pp wave from* $N = 4$ *super Yang Mills*, `hep-th/0202021`.

[9] C. Hull, JHEP 9807, 021 (1998), `hep-th/9806146`;
C. Hull, *Duality and the Signature of Space-Time*, JHEP 9811, 017 (1998), `hep-th/9807127`.

[10] A. Sen, *Remarks on Techyon Driven Cosmology*, `hep-th/0312153` and references therein.

[11] A. Dabholkar, G. Gibbons, J. Harvey and F. Ruiz-Ruiz, Nucl. Phys. **B 340** (1990) 53.

[12] S. Kachru, R. Kallosh, A. Linde and S. Trivedi, *de Sitter vacua in string theory*, Phys. Rev. D68 (2003) 046005, `hep-th/0301240`;
S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and and S. Trivedi, *Towards inflation in string theory*, `hep-th/0308055`.

[13] H. Singh, *Generalised Penrose Limits and PP-Waves*, Phys. Lett. **B583** (2004) 315, `hep-th/0307088`.

[14] J.T. Liu, W.A. Sabra and W.Y. Wen, *Consistent reductions of IIB***/M** *theory*, JHEP 0401, 007 (2004) `hep-th/0304253`.

[15] G. Gibbons and C. Hull, *De Sitter Space from Warped Supergravity Solutions*, `hep-th/0111072`.

[16] G. T. Horowitz and A. R. Steif, *Spacetime singularities in string theory*, *Phys. Rev. Lett.* **64** (1990) 266.

[17] E. Witten, *Quantum Gravity In De Sitter Space*, `hep-th/0106109`; M. Spradlin, A. Strominger and A. Volovich, Les Houches Lectures on De Sitter Space, `hep-th/0110007`.